Math 129 - 011

Problem Session Notes

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So, in class we were working on the problem

$$\int \frac{\tan 3x}{\cos(3x)+4}\,dx.$$

And you'll recall the thing to do was to first write the tangent in terms of sines and cosines, as follows

$$\int \frac{\sin 3x}{\cos 3x(\cos(3x)+4)}\,dx,$$

and then use a u-substitution. It was pointed out the u-substitution could be done in two ways: $u=\cos 3x$ or $u=\cos(3x)+4$. We will see that either way will work. If we take the first, we get

$$\begin{aligned}\int \frac{\sin 3x}{\cos 3x(\cos(3x)+4)}\,dx &= -\frac{1}{3}\int \frac{1}{u(u+4)}\,du && (1)\\ &= -\frac{1}{3}\int \frac{1}{(u-0)(u-(-4))}\,du. && (2)\end{aligned}$$

Which is a perfect candidate for entry 26 on our table. For which, we get

$$\begin{aligned}\int \frac{\tan 3x}{\cos(3x)+4}\,dx &= -\frac{1}{3}\int \frac{1}{(u-0)(u-(-4)}\,du \qquad a=0 \text{ and } b=-4 && (3)\\ &= -\frac{1}{3}\left[\frac{1}{0-(-4)}\big(\ln|u-0|-\ln|u-(-4)|\big)+C\right] && (4)\\ &= -\frac{1}{12}\ln|u|+\frac{1}{12}\ln|u+4|+C && (5)\end{aligned}$$

This is, of course, not the final answer. We need to back substitute x. We get then

$$\begin{aligned}\int \frac{\tan 3x}{\cos(3x)+4}\,dx &= -\frac{1}{12}\ln|u|+\frac{1}{12}\ln|u+4|+C && (6)\\ &= -\frac{1}{12}\ln|\cos 3x|+\frac{1}{12}\ln|\cos(3x)+4|+C && (7)\end{aligned}$$

If we use the other u-sub we get

$$\begin{aligned}\int \frac{\tan 3x}{\cos(3x)+4}\,dx &= \int \frac{\sin 3x}{\cos 3x(\cos(3x)+4)}\,dx && (8)\\ &= -\frac{1}{3}\int \frac{1}{(u-4)u}\,du \qquad a=4 \text{ and } b=0 && (9)\\ &= -\frac{1}{3}\left[\frac{1}{4-0}\big(\ln|u-4|-\ln|u|\big)+C\right] && (10)\\ &= -\frac{1}{12}\ln|u-4|+\frac{1}{12}\ln|u|+C && (11)\\ &= -\frac{1}{12}\ln|\cos 3x|+\frac{1}{12}\ln|\cos(3x)+4|+C && (12)\end{aligned}$$

Exactly the same thing after we put it back in terms of x.

QUESTION: The answer we got equivalent to

$$\frac{1}{12}\ln|1+4\sec(3x)|+C?$$

ANSWER: Yes