Math 129 - 011

Problem Session Notes

March 29, 2021

Jared McBride

A student pointed out to this to me at the end of lecture. And it is something I would like us all to understand because it is a more simple technique than what I have taught you earlier.

When summing a geometric series all that maters is the first term and the common ratio. So, if I have a goemetric series (infinite sum) and its **first term** is a and it's **common ratio** is r then the sum is

$$\frac{a}{1-r}$$

provide $|r| < 1$.

Examples

1. $5\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n = \frac{5}{1-\frac{1}{5}} = \frac{25}{4}$
2. $5\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n = \frac{5\left(\frac{1}{5}\right)^2}{1-\frac{1}{5}} = \frac{1}{4} \quad \left(= \frac{25}{4} - 5 - 1\right)$
3. $x^5\sum_{n=0}^{\infty}\frac{\sin x}{x}\left(\frac{5}{x^2}\right)^n = \frac{x^4 \sin x}{1-\frac{5}{x^2}} = \frac{x^6 \sin x}{x^2-5}$ provided $|x| > \sqrt{5}$.
4. $x^5\sum_{n=2}^{\infty}\frac{\sin x}{x}\left(\frac{5}{x^2}\right)^n = \frac{25 \sin x}{1-\frac{5}{x^2}} = \frac{25x^2 \sin x}{x^2-5}$

In summary, for geometric series, you **do not need to worry about the indices** You only need the first term (*regardless of what n equals*) and the common ratio. This is different from what I lead you to believe in class. Though what I demonsrtated is not wrong, and important for other summation formulas, it is not the best way to find geometric sums that don't start at $n = 0$.