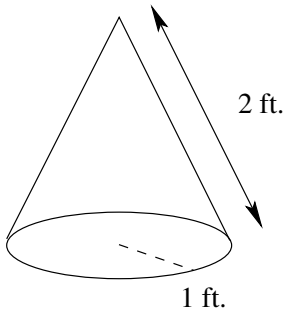
Worksheet 8

1. The right circular cone shown in the figure below has radius 1 foot and lateral height 2 feet. In each of the following problems, the cone has non-uniform density. Use the information given to determine the mass of the cone in each case.



(a) The density of the cone at any point is proportional to the distance from that point to the central axis of the cone (the axis from the apex to the center of the base). The density of the cone at the circumference of the base is 4 pounds per cubic foot.

(b) The density of the cone at any point is proportional to the distance from that point to the apex of the cone. The density of the cone at the circumference of the base is 4 pounds per cubic foot.

2. Consider the surfaces $z = x^2 + y^2 - 7$ and $z = 9 - 3y^2$.

   (a) Sketch the two surfaces.

   (b) Describe the curve in which the two surfaces intersect.

   (c) There is a bounded region in three-dimensional space that is bounded by these two surfaces. Set up, but do not evaluate, an integral representing the volume of this region.

3. For each of the following descriptions of a particle's motion, write a formula for a parametric function $\vec{r}(t)$ that describes the particle's motion, including limits on t when appropriate. Assume that t is in seconds.

   (a) The particle travels in a straight line from the point $(0, -2, 3)$ to the point $(4, -4, 2)$.

   (b) The particle travels along the same path it did in part (a), but does so in exactly five seconds.

   (c) The particle travels clockwise around the circle in the xy-plane with center $(3, -4)$ and radius 5, making a full revolution every three seconds.

   (d) The particle travels along the curve $x - y^2 = 4$, passing through the point $(13, 3)$ at time $t = 7$.

4. A boy with a slingshot stands at the origin and sees a paper airplane flying in a straight line. The position of the paper airplane at time t is given by the equations

$$x(t) = 6 - 4t, \quad y(t) = 2 + 2t,$$

where t is in seconds and x, y are in feet. The boy wants to fire a rock from the slingshot and hit the paper airplane. He will release the rock at the moment the paper airplane passes through the point $(6, 2)$. Answer the following questions:

   (a) If the boy fires the rock and hits the airplane, and the y-component of the rock's velocity is 12 feet per second, in what direction did the boy fire the rock?

   (b) If the boy wants to fire the rock and hit the airplane, but he can only fire the rock at a speed of 10 feet per second, in what direction should he fire the rock?

5. The position function of a particle in space is $\vec{r}(t) = (t^2 - 4)\vec{i} + (12t - t^3)\vec{j} + (5t)\vec{k}$.

   (a) What is the speed of the particle at time $t = 0$?

   (b) Find an equation for the plane perpendicular to the path of the particle at the point $(-3, 11, 5)$.

   (c) A second particle travels so that its position is always the reflection of the first particle's position across the xz-plane. Give the position function of this second particle.

   (d) Let C be the path of the first particle from $t = 0$ to $t = 10$, and let $\vec{F}$ be the vector field defined by $\vec{F}(x, y, z) = -x\vec{i} - y\vec{j} - z\vec{k}$. Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$.

6. A 160-lb man carries a 25-lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. The silo is 90 ft high and the man makes exactly three complete revolutions to get to the top.

   (a) How much work is done by the man against gravity in climbing to the top?

   (b) Now suppose he makes the same trip again, this time his bucket leaks at a rate of 5-oz every second. If he hurries up the stairs at a velocity of 5 ft/s, how much work is done? (How did his hurrying affect the work being done?)