Worksheet 3

1. During the winter season in Tucson, people across the city use electric heating to keep their homes warm. Let $C(A, T)$ be the daily cost in dollars of keeping an apartment whose floor area is a total of A square feet heated to a temperature of 75°F, provided that the outdoor temperature is T degrees Fahrenheit. Answer the following questions:

   (a) What does $C_A(600, 60)$ represent? What are its units?

   (b) Which of the following possible values for $C_T(800, 50)$ is the most reasonable: -10, -0.1, 0, 0.1, or 10? Explain your answer in terms of the context.

   (c) Assume $C_T(800, 50)$ takes the value you choose in (b). Given $C(800, 50) = 1.2$, estimate $C(800, 48)$.

   (d) Decide whether $C_T(550, 60)$ is greater than, equal to, or less than $C_T(710, 60)$. Explain your answer in terms of the context.

2. A research and development lab for a pharmaceutical company ($$$) studies the use of bacteria injected with particular mRNA to produce pharmaceuticals. A certain culture of bacteria has been studied experimentally and a model has been purposed for it's growth under various amounts of two media (bacteria food). Specifically, with an initial population of 1000 bacteria, the time it takes for the population to double is $t(a, b)$ in minutes where a is the amount of medium A and b is the amount of medium B both in grams.

   (a) What does $t_a(0.3, 0.7)$ represent? What are its units?

   (b) Suppose $t(0.7, 1.3) = 4.33$, $t_a(0.7, 1.3) = .2$ and $t_b(0.7, 1.3) = -.1$. What would be your best approximation for the value of $t(0.75, 1.2)$?

   (c) Useing the information from the previous part, you would like to adjust the amounts of media in an optimal way. Let Δa and Δb be the amounts by which you purpose to adjust $a(= 0.7)$ and $b(= 1.3)$, respectively. At what ratio $(\Delta a/\Delta b)$ should your adjustments be to optimize the bacteria growth.

   (d) Time is money and media is not free. If on average the time it takes to culture the bacteria costs \$92.30 per minute and the cost of media A and B are, respectively, \$8.65 and \$13.35 per kg, given the information above, what is the cost efficient ratio $(\Delta a/\Delta b)$ for adjusting the media for the bacteria.

3. In each case below, give an example of:

   (a) A non-linear function $f(x, y)$ such that $f_x(0, 0) = 2$ and $f_y(0, 0) = 3$.

   (b) Functions $f(x, y)$ and $g(x, y)$ such that $f_x = g_x$ but $f_y \neq g_y$.

   (c) A non-constant function $f(x, y)$ such that $f_x = 0$ everywhere.

   (d) A function $f(x, y)$ with $f_x > 0$ and $f_y < 0$ everywhere.

   (e) A function $f(x, y)$ with $f_x(x, y) = y$ and $f_y(x, y) = x$.

4. Consider the function $f(x, y) = y - x^2$.

   (a) Plot the level curves of the function for $z = -2, -1, 0, 1, 2$.

   (b) Imagine the surface whose height above any point (x, y) is given by $f(x, y)$. Suppose you are standing on the surface at the point where $x = 1, y = 2$.

      i. What is your height?

      ii. If you start to move on the surface parallel to the y-axis in the direction of increasing y, does your height increase or decrease?

      iii. Does your height increase or decrease if you start to move on the surface parallel to the x-axis in the direction of increasing x?

      iv. Are you ascending or descending when you go from $(1, 2)$ in the direction making an angle of $3\pi/4$ with the x-axis?

      v. Suppose the units of length are in meters and you start to move in the direction you found in part 4 at a rate of 3 meters per sec. At what rate is your height changing with respect to time?

5. Consider the function $f(x, y) = -61 - 9x^2 + 54x - 4y^2 + 16y$.

   (a) Plot the level curves of the function for $z = -108, 0, 18, 36$.

   (b) Imagine the surface whose height above any point (x, y) is given by $f(x, y)$. Suppose you are standing on the surface at the point where $x = 2, y = 3$.

      i. What is your height?

      ii. If you start to move on the surface parallel to the y-axis in the direction of increasing y, does your height increase or decrease?

      iii. Does your height increase or decrease if you start to move on the surface parallel to the x-axis in the direction of increasing x?

      iv. Are you ascending or descending when you go from $(1, 2)$ in the direction making an angle of $3\pi/4$ with the x-axis?

      v. Suppose the units of length are in meters and you start to move in the direction you found in part 4 at a rate of 3 meters per sec. At what rate is your height changing with respect to time?

6. Let $f(x, y)$ be a differentiable function of two variables such that $f(-6, 11) = 14$, $f_x(-6, 11) = 3$, and $f_y(-6, 11) = -2$. Based on this information, estimate the following:

   (a) The value of $f(-6, 11.4)$.

   (b) The value of $f(-6.2, 11)$.

   (c) The value of $f(-5.7, 10.9)$.

   (d) The slope of the contour $f = 14$ at the point $(-6, 11)$.

   (e) The value of $f(x, y)$, where (x, y) is a point near $(-6, 11)$.