Worksheet 2

1. Two functions $f(x, y)$ and $g(x, y)$ are given below. For each of the following statements, decide whether the statement is true for each function f and g. Record your answers in the table shown below.

$$f(x, y) = xy^2 \qquad g(x, y) = x^2 + y + 4$$

(a) This function is defined for all values of x and y.

(b) This function is negative for some values of x and y.

(c) The cross-sections of the graph of this function parallel to the xz-plane are lines.

(d) The cross-sections of the graph of this function parallel to the yz-plane are parabolas.

(e) The contours of the graph of this function are parabolas.

(f) The contours of the graph of this function are ellipses.

(g) The contours of the graph of this function are hyperbolas.

(h) The graph of this function intersects the x-axis.

	$f(x, y)$	$g(x, y)$
(a)		
(b)		
(c)		
(d)		
(e)		
(f)		
(g)		
(h)		

2. Write an equation for each of the following planes.

(a) A plane that passes through the point $(24, -13, 2)$ and is parallel to the xz-plane.

(b) The plane $z = f(x, y)$, where $f(x, y)$ is a linear function with the following partial table of values:

$y \backslash x$	20	25	30	35
35		23		
30	26		14	
25				5
20			8	

(c) A plane that passes through the point $(-3, 0, 2)$ but does not intersect the plane $x - 2y + 5z = 14$.

3. Two functions $p(x, y)$ and $q(x, y)$ are given below. For each of the following statements, decide whether the statement is true for each function f and g. Record your answers in the table shown below.

$$p(x, y) = 4x^2 + 8x + y^2 \qquad q(x, y) = \frac{1 + x^2}{1 + y^2}$$

   (a) This function is defined for all values of x and y.

   (b) This function is negative for some values of x and y.

   (c) The cross-sections of the graph of this function parallel to the xz-plane are lines.

   (d) The cross-sections of the graph of this function parallel to the yz-plane are parabolas.

   (e) The contours of the graph of this function are parabolas.

   (f) The contours of the graph of this function are ellipses.

   (g) The contours of the graph of this function are hyperbolas.

   (h) The graph of this function intersects the x-axis.

	$p(x, y)$	$q(x, y)$
(a)		
(b)		
(c)		
(d)		
(e)		
(f)		
(g)		
(h)		

4. Write an equation for each of the following planes. In some cases, there may be more than one plane satisfying the given criteria.

   (a) A plane that passes through the point $(-3, 0, 2)$ and is perpendicular to the plane $x - 2y + 5z = 14$.

   (b) A plane with z-intercept 12 that contains the line $x - 2y = 4$ in the xy-plane.

   (c) A plane containing the triangle with vertices $(1, 0, -2)$, $(2, 2, 1)$, and $(-1, 2, 4)$.

5. Sketch the graph of each of the following equations in $\mathbb{R}^3$:

   (a) $y = x^2 + z^2$

   (b) $z = \sqrt{x^2 + y^2}$

   (c) $z = \sqrt{(x - 3)^2 + y^2}$

   (d) $z = \sqrt{4 - (x^2 + y^2)}$

6. Sketch the graph of each of the following equations in $\mathbb{R}^3$:

   (a) $z = x^2 + y^2$

   (b) $16 = x^2 + z^2$

   (c) $z^2 = x^2 + y^2$

   (d) $z = 4 - \sqrt{x^2 + y^2}$