Worksheet 12

1. Decide whether each of the following statements is true or false. Explain your answers.

   (a) If a vector field $\vec{F}$ in 3-space has zero divergence, then $\vec{F} = a\vec{i} + b\vec{j} + c\vec{k}$ where a, b, c are constants.

   (b) The flux of any constant vector field through any closed, smooth, oriented surface is zero.

   (c) If S is an oriented surface in 3-space, and $-S$ is the same surface, but with the opposite orientation, then
   $$\int_S \vec{F} \cdot d\vec{A} = -\int_{-S} \vec{F} \cdot d\vec{A}.$$

   (d) If $\vec{F}(x, y, z) = -y^2\vec{k}$, and S is the surface of the cube of edge length 2 with vertices $(\pm 1, \pm 1, \pm 1)$ oriented outward, then the only faces of $\vec{F}$ through which the flux of $\vec{F}$ is nonzero are the top and bottom faces

   (e) If $\vec{F}$ is a 3-dimensional vector field such that $\vec{F} = \text{grad}(f)$ for some differentiable function f, then $\text{div}\vec{F} = 0$.

2. Consider the cylinder of radius 3 about the y-axis with $0 \leq y \leq 8$. Let S be the curved lateral surface of this cylinder together with the base of the cylinder that lies in the xz-plane, oriented inward. Let $\vec{F}(x, y, z) = (-2yz)\vec{i} + xz\vec{j} + y^2\vec{k}$. Compute
   $$\int_S \text{curl}\vec{F} \cdot d\vec{A}.$$

3. Let S be the paraboloid $z = x^2 + y^2 - 5$ closed off by the disk $x^2 + y^2 \leq 9$ on the plane $z = 4$, oriented outward. Let $\vec{F}(\vec{r}) = \vec{r}$. Compute $\int_S \vec{F} \cdot d\vec{A}$.