Worksheet 11

1. Suppose that $\vec{F}(x, y, z) = (3x - y)\vec{i} - y^2 z\vec{j} + z^2\vec{k}$. Several small surfaces are given below. For each surface, predict whether the flux of $\vec{F}$ through the given surface is positive, negative, or zero. Try to do this without computing any integrals.

   (a) The sphere of radius 0.2 centered at the point $(-2, 2, 1)$.

   (b) The surface of the cube $0.4 \leq x \leq 0.7$, $4.8 \leq y \leq 5.1$, $-1.5 \leq z \leq -1.2$, oriented inward.

   (c) The circular disc of radius $1/\pi$ in the xy-plane centered at the point $(1, 4)$, oriented upward.

   (d) The square with vertices $(1, 8, 3)$, $(1, 9, 3)$, $(1, 9, 2)$, and $(1, 8, 2)$, oriented frontward.

2. Let S be the sphere of radius 2 centered at the origin, oriented outward. Let $\vec{F}(x, y, z) = 4z\,\vec{i} - y\,\vec{j} + z^2\,\vec{k}$.

   (a) Consider the three components of $\vec{F}$. Determine whether each component makes a positive, negative, or zero contribution to the flux of $\vec{F}$ through S.

   (b) Compute $\int_S \vec{F} \cdot d\vec{A}$.

3. Compute each of the following:

   (a) The flux of the vector field $\vec{F}(x, y, z) = 4\,\vec{i} - \vec{k}$ through the rectangle with vertices $(0, 0, 4)$, $(3, 0, 4)$, $(3, 6, 1)$, and $(0, 6, 1)$, oriented upward.

   (b) The flux of the vector field $\vec{F}(x, y, z) = -z^2\,\vec{i} + (x^2 y + yz^2)\,\vec{j} + 3xy\,\vec{k}$ through the circular disc of radius 4, centered at the point $(0, 3, 0)$ and parallel to the xz-plane, oriented rightward.

   (c) The flux of the vector field $\vec{F}(x, y, z) = 2y\,\vec{i} - 2x\,\vec{j} + \vec{k}$ through the part of the surface $y^2 + z^2 = 9$ satisfying $0 \leq x \leq 8$, $y \geq 0$, and $z \geq 0$, oriented towards the x-axis.