# Worksheet 1

1. The superheroine Differentia is battling her arch-nemesis Vek-Tor for control of the city of Metropolis. Hoping to destroy Vek-Tor from the sky, Differentia takes off southward from the center of the city at an angle of 35° with the ground and flies a distance of 720 meters to a point $P$. Hoping to avoid her, Vek-Tor takes off eastward from the same spot at an angle of 28° with the ground and flies a distance of 380 meters to a point $Q$. (See Figure 1.)

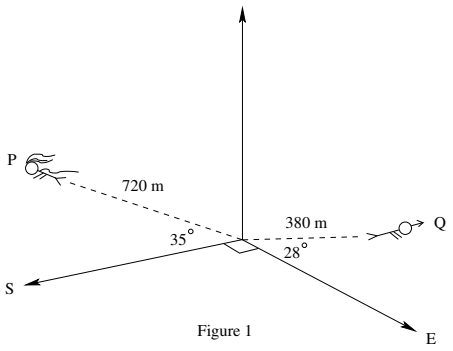


Figure 1

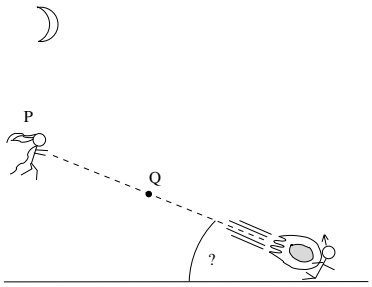


Figure 2

   (a) Once Differentia and Vek-Tor have reached points $P$ and $Q$, respectively, how far apart are they?

   (b) Differentia aims a huge fireball at Vek-Tor and blasts him with such great force that the fireball drives him backwards until he hits the ground. (See Figure 2.) At what angle does Vek-Tor crash into the ground?

2. Find two vectors $\vec{v}$ and $\vec{w}$, in component form, having the following properties. There is more than one possible answer.

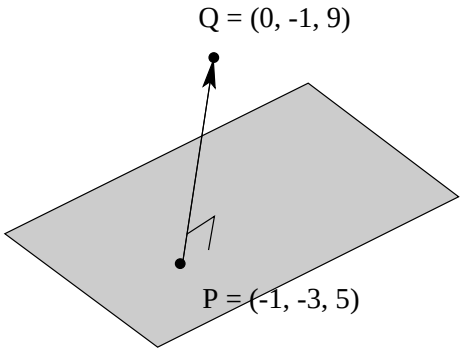
   - $\vec{v} \cdot \vec{w} = -20$
   - $\vec{v} \times \vec{w} = 15\vec{j}$
   - $\vec{v}$ and $\vec{w}$ have the same length.
   - The $\vec{i}$-component of $\vec{v} + \vec{w}$ is negative.

3. A hungry falcon spots a pizza delivery person on a bike 300 meters due east of its nest, which is very close to the ground. The falcon hatches a plan to steal a pizza from the unsuspecting delivery person, who starts bicycling at a velocity of 6 meters per second in a direction 45 degrees east of north. The falcon can fly at a constant speed of 20 meters per second in any direction.

   (a) If the falcon starts flying the moment the bicycle starts moving, in what direction should it fly so that it intercepts the delivery person (and a free meal)?

   (b) How long will it take for the falcon to reach the bicycle?

4. The picture below shows a plane that contains the point $P = (-1, -3, 5)$. The plane is perpendicular to the vector $\overrightarrow{PQ}$, where $Q = (0, -1, 9)$.




(a) Let $R = (1, 0, 3)$. Does the vector $\overrightarrow{PR}$ lie in the plane shown? If not, what is the angle between $\overrightarrow{PR}$ and the plane?

(b) Let $S = (-2, 1, 2)$. Does the vector $\overrightarrow{PS}$ lie in the plane shown? If not, what is the angle between $\overrightarrow{PS}$ and the plane?

(c) Suppose we are given a point $T = (x, y, z)$. Describe a general method for determining whether $\overrightarrow{PT}$ lies in the plane shown.

(d) The plane in the picture divides space into two half-spaces, one containing $Q$ and one not containing $Q$. Suppose that $T = (x, y, z)$ is a point that is not in the plane. How can we decide which half-space $T$ is in?

5. Suppose that $\vec{a}$, $\vec{b}$, and $\vec{c}$ are nonzero three-dimensional vectors. Fill in the table below. In the firt column determine which of the expression makes sense. (Assume that the symbol $\cdot$ always means dot product, not scalar multiplication or multiplication of numbers.) For each expression that does make sense, decide whether the value of the expression is a scalar or a vector. Also decide whether the value of the expression is zero or nonzero in each of the following situations:

(i) $\vec{a}$, $\vec{b}$, and $\vec{c}$ all point in the same direction;

(ii) $\vec{a}$, $\vec{b}$, and $\vec{c}$ all lie in the same plan; and

(iii) $\vec{a}$, $\vec{b}$, and $\vec{c}$ are all perpendicular to one another.

| Expresion | Makes sense? | Vector or scalar? | Zero or nonzero under (i)? | Zero or nonzero under (ii)? | Zero or nonzero under (iii)? |
|---|---|---|---|---|---|
| $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ | | | | | |
| $(\vec{a} \times \vec{b}) \cdot \vec{c}$ | | | | | |
| $(\vec{a} \cdot \vec{b}) \times \vec{c}$ | | | | | |
| $(\vec{a} \times \vec{b}) \times \vec{c}$ | | | | | |

6. The diagonals of a parallelogram are given by $\vec{a} = 3\vec{i} - 4\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} - 6\vec{k}$. Show that the parallelogram is a rhombus (all sides are equal) and give the length of its sides and the measure of its angles. Also find it's area.