Major Theorems Cheat Sheet

Five versions of the fundamental theorem of calculus. Each involve a set, it's boundry, a function and some sort of derivative.

Theorem	Conditions	Result	When do I use it...
Fundamental Theorem of Calculus	• F is differentiable from a to b	$\int_{[a,b]} F'(x)\,dx = F(b) - F(a)$	• Evaluating integrals in a single variable
Fundamental Theorem for Line Integrals	• C is piecewise smooth starting at P and ending at Q • grad f is continuous on C	$\int_C \text{grad } f \cdot d\vec{r} = f(Q) - f(P)$	• Evaluating "open" line integrals in a conservative field
Green's Theorem (2-dimensional Stokes')	• R is a oriented region with piecewise smooth oriented boundary C • $\vec{F} = F_1\vec{i} + F_2\vec{j}$ is smooth in an open region containing R and C	$\int_R \left(\frac{\partial F_1}{\partial x} - \frac{\partial F_2}{\partial y}\right) dA = \int_C \vec{F} \cdot d\vec{r}$	• Evaluating closed line integrals in a non-conservative field in 2-dimensions • Evaluating circulation (in 2-dimensions)
Stokes' Theorem (higher dimensional Green's)	• S is a smooth oriented surface with piecewise smooth oriented boundary • $\vec{F}$ is smooth in an open region containing S and C	$\int_S \text{curl } \vec{F} \cdot d\vec{A} = \int_C \vec{F} \cdot d\vec{r}$	• Evaluating closed line integrals in a non-conservative field in 3-dimensions • Evaluating circulation in 3-dimensions
Divergence Theorem	• W is a solid region whose boundary S is a piecewise smooth surface with outward orientation • $\vec{F}$ is smooth in an open region containing W and S	$\int_W \text{div } \vec{F}\, dV = \int_S \vec{F} \cdot d\vec{A}$	• Evaluating flux of a field through a *closed* surface • Can be used to find flux through an "open" surface by first closeing the surface finding the flux through that and then subtracting the flux through what you added to close the surface.

Notice the pattern integral of derivative over set = integral of function over boundry.