Week of November 12

Instructions: *Please work on* **Problems 1 and 2 only**, *in advance, as much as possible. Please also think about Problem 3 (though you will not be graded on it). You will work with your peers on Problem 3 in class. Come ready to describe, write up, and make further progress with others the on work you have done. Work hard, enjoy!*

1. Consider the vector field $\vec{G}(x, y) = (x^2y)\vec{i} + (3xy^2)\vec{j}$. Let C_1 be the oriented line segment from $(0, 0)$ to $(2, 0)$, let C_2 be the sequence of line segments from $(0, 0)$ to $(1, 1)$ and from $(1, 1)$ to $(2, 0)$, let C_3 be the semicircle from $(0, 0)$ to $(2, 0)$ passing through $(1, 1)$, and let C_4 be the sequence of line segments from $(0, 0)$ to $(0, -3)$, from $(0, -3)$ to $(2, -3)$, and from $(2, -3)$ to $(2, 0)$. Rank these four paths according to the value of the line integral of $\vec{G}$ along each path. If possible, try to do this without actually computing any integrals.

2. (a) Describe, in words, the motion of a particle moving through the following paths. Sketch also a graph of each of the physical paths.
   C_1 : $\vec{r}(t) = t\cos(2\pi t)\,\vec{i} + t\sin(2\pi t)\,\vec{k}, 0 \le t \le 2$
   C_2 : $\vec{r}(t) = t\cos(2\pi t)\,\vec{i} + t\,\vec{j} + t\sin(2\pi t)\,\vec{k}, 0 \le t \le 2$

   (b) Evaluate $\int_{C_2} \vec{F} \cdot d\vec{r}$, for the vector field $\vec{F} = yz\,\vec{i} + z(x + 1)\,\vec{j} + (xy + y + 1)\,\vec{k}$.

   (c) Find a non-zero vector field $\vec{G}$ such that:

   $$\int_{C_1} \vec{G} \cdot d\vec{r} = \int_{C_2} \vec{G} \cdot d\vec{r},$$

   where C_1, C_2 are the curves in Problem (a) above. Explain briefly how you reasoned to find $\vec{G}$.

   (d) Find two different, non-zero vector fields $\vec{H}_1$, $\vec{H}_2$ such that:

   $$\int_{C_1} \vec{H}_1 \cdot d\vec{r} = \int_{C_1} \vec{H}_2 \cdot d\vec{r}$$

   where C_1 is the curve in Problem (a) above. Explain briefly how you reasoned to find the two fields.

3. Evaluate the following line integrals. For each problem, try to come up with the most efficient strategy for computing the integral. If you use a theorem, be sure to check that all the hypotheses of the theorem hold!

   (a) The line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = y\vec{i} - x\vec{j}$, and C is the semicircular arc from $(0, 0)$ to $(0, 4)$ passing through the point $(2, 2)$.

   (b) The line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$, and C is the upward oriented spiral that starts at the point $(3, 0, 0)$ and ends at the point $(3, 0, 8)$, wrapping around the z-axis twice during this interval. (The spiral, when collapsed onto the xy-plane, is a perfect circle centered around the origin.)

   (c) The line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = 2\vec{i} + x^2\vec{j}$, and C is the oriented path consisting of three line segments: from $(0, 0)$ to $(0, 4)$, from $(0, 4)$ to $(4, 6)$, and from $(4, 6)$ to $(4, 0)$.

   (d) The line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = (\cos x \sin y)\vec{i} + (\sin x \cos y)\vec{j}$, and C is the part of the parabola $y = 10 - x^2$ that lies above the line $y = 1$, oriented from left to right.

   (e) The circulation $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = 5y\vec{i} + x^2\vec{j}$, and C is the oriented square with vertices $(2, 0)$, $(0, -2)$, $(-2, 0)$, and $(0, 2)$, in that order.

   (f) The circulation $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = z\vec{i} + 2z\vec{j}$, where C is the parametrized curve $\vec{r}(t) = (2\cos t)\vec{i} + (3\sin t)\vec{j} - 4\vec{k}$ for $0 \le t \le 2\pi$.